Comparison of two forward solutions approaches in Lorentz Force Evaluation

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Two different forward models for Lorentz force evaluation, the approximate forward solution (AFS) and the novel extended area approach (EAA), are compared using a goal function scan. A laminated aluminum specimen that contains a cuboidal defect of the size 12 mm × 2 mm × 2 mm at 2 mm depth is simulated by the finite element method. Both methods are applied for defect reconstruction and showed a correct depth estimation with normalized root mean square errors (NRMSE) of 6.05 % for AFS and 1.67 % for EAA, respectively. The EAA yields defect dimensions of 11 mm × 2 mm × 2 mm, whereas AFS determines 7 mm × 10 mm × 2 mm.

Index Terms—Eddy current, Lorentz force evaluation, inverse problem, nondestructive evaluation

I. INTRODUCTION

The Lorentz force evaluation (LFE) is a nondestructive method, which reconstructs defects from perturbations in Lorentz forces that act on a permanent magnet, which moves relatively to a conductive specimen. Previous defect reconstructions have been performed by truncated singular value decomposition [1], differential evolution [2] and current density reconstruction [3], where all were based on the approximate forward solution (AFS) [1]. Recently, the more accurate extended area approach (EAA) has been introduced as forward model for LFE [4]. It is the aim of the current study to compare the performance of AFS and the novel EAA with regard to defect reconstruction performance. For that purpose, a goal function scan [5] is applied to a simulated dataset obtained by the finite element method (FEM), which is used for modeling a specimen consisting of stacked aluminum sheets and a cuboidal defect.

![Fig. 1. Benchmark problem: A package of aluminum sheets with a cuboidal defect is moved relatively to the spherical permanent magnet, where the interaction of the induced eddy currents (orange lines) with the magnetic field leads to Lorentz forces; the figure is not to scale for better visualization.](image)

Permanent magnet Remanence \( B_r = 1.17 \) T

Defect, \( \sigma_d = 0 \) S/m

Aluminum sheets, \( \sigma_0 = 30.61 \) MS/m

Permanent magnet with the velocity \( v = 0.01 \) m/s (Fig. 1). Due to the relative movement eddy currents are induced, where the interaction with the magnetic field leads to Lorentz forces. In presence of a defect these eddy currents are perturbed (Fig. 1) and so are the Lorentz forces. The specimen consists of stacked aluminum sheets, where each sheet possesses a thickness of \( \Delta z = 2 \) mm. The spherical permanent magnet with the homogenous magnetization \( M \) is located at the lift-off distance \( d = 1 \) mm above the top surface of the specimen. It is characterized by a remanence of \( B_r = 1.17 \) T and a diameter \( D_m = 15 \) mm. A cuboidal defect with the conductivity \( \sigma_d = 0 \) S/m and \( L_d \times W_d \times H_d = 12 \) mm × 2 mm × 2 mm is located at the depth \( d = 2 \) mm.

B. Approximate Forward Solution and Extended Area Approach

The spherical permanent magnet is modelled as a magnetic dipole located at its center \( r_0 \). The magnetic flux density at the point \( r \) can be calculated by

\[
B_r = \frac{\mu_0}{4\pi} \left( 3 \frac{m \cdot (r - r_0)}{|r - r_0|^5} - \frac{m}{|r - r_0|^3} - \frac{m}{|r - r_0|^3} \right),
\]

where \( m \) denotes the magnetic moment. Due to the small velocity \( v \), the secondary magnetic field from the induced eddy currents can be neglected. Thus, the weak reaction approach can be applied [6].

The defect response signal (DRS) \( \Delta F \) [4], which forms the basis for the defect reconstruction is defined by

\[
\Delta F = \int_{V_d} \left( j - j_0 \right) \times B_r dV - \int_{V_d} j_0 \times B_r dV,
\]

where \( j \) and \( j_0 \) represent the current density in the specimen with and without a defect, respectively. The volumes of the specimen and the defect are denoted by \( V \) and \( V_d \).

II. METHODS

A. Benchmark problem

A specimen with \( L \times W \times H = 400 \) mm × 400 mm × 100 mm and the conductivity \( \sigma_0 = 30.61 \) MS/m is moved relatively to the permanent magnet with the velocity \( v = 0.01 \) m/s (Fig. 1).
The AFS neglects the first term of (2), which means that only the
defect as a fictitious conducting region is taken into account
for the calculation of the DRS. The defect is discretized into \( K \)
volume elements (voxels) of volume \( V_E \). The DRS is then
approximated by [1]
\[
\Delta F_{\text{AFS}} = V_E \sum_{k=1}^{K} \Delta j_k \times B_k.
\]  
(3)
The distortion current density \( \Delta j_k = j_0 \) can be calculated by
Ohm’s law for moving conductors \( \Delta j_k = \sigma_0 \left( -\nabla \phi_k + v \times B_k \right) \) [1], where \( B_k \) denotes the magnetic flux densities inside the
voxels. The EAA extends the region for forward calculation in
\( x \)- and \( y \)-direction around the defect, which approximates the
first term of (2). The extended area is discretized into \( E \) cuboidal
voxels. Thus \( \Delta F \) is approximated by [4]
\[
\Delta F_{\text{EAA}} = V_E \sum_{k=1}^{E} \Delta j_k \times B_k + \Delta F_{\text{AFS}},
\]  
(4)
where the magnetic flux densities inside the extended voxels are
denoted by \( B_k \). The distortion current densities \( \Delta j_k \) in the
outer voxels can be determined by [4]
\[
\Delta j_k = \frac{C_d}{2 \pi L_k} \sum_{x=1}^{V_E} \left[ \frac{2 \Delta j_k \cdot (r_x - r_k)}{|r_x - r_k|^4} \right],
\]  
(5)
where the dipolar correction factor \( C_d = 1 + (\pi/4)(L_d/W_d) \) holds
for cuboidal defects [4]. The position vectors of the voxels’ cen-
troids in the defect and the extended region are denoted by \( r_k \)
and \( r_x \), respectively.

For the EAA, the decision of an appropriate expansion size
is important. The DRS \( \Delta F_{\text{EAA}} \) is calculated for the benchmark
problem for the extensions \( x \in [0, 1, 2, \ldots, 7] \) in \( x \)- and \( y \)-direction, where 0 means AFS is applied. The expansion
has been chosen to \( x = 5 \times \max(W_a, L_d) \) as the adapted
normalized root mean square error (aNRMSE)—stopped
improving.

C. Goal Function Scan

The DRS is calculated by AFS according to (3) and by EAA
according to (4), respectively, for the combinations of
\( L_d = [1, 2, \ldots, 50] \) and \( W_d = [1, 2, \ldots, 50] \) from the \( 1^{\text{st}} \)
to the \( 11^{\text{th}} \) layer. For each calculation, the aNRMSE is determined
as a goal function value by
\[
a\text{NRMSE} = \frac{1}{7} \sum_{l=1}^{7} \left[ \frac{1}{n} \sum_{i=1}^{n} \left( \Delta F_{\text{AFS}} - \Delta F_{\text{EAA}} \right) \right] \],
\]  
(6)
where \( n \) indicates the current position of the magnetic dipole.
Only the \( x \)- and \( z \)-error components are used since the \( y \)-
component of the DRS shows too large errors for both forward
models. For every aluminum layer, the \( L_d \times W_d \)-combination
with the lowest aNRMSE is determined for AFS and EAA. The
layer with the lowest aNRMSE gives the result for the depth and
the size of the defect produced by the goal function scan for
AFS and EAA.

III. RESULTS AND DISCUSSION

Fig. 2 shows the aNRMSE and the corresponding estimated
defect extensions \( L_d \times W_d \) in \( x \)- and \( y \)- direction in mm²
(indices, Fig.2) over the aluminum layers for AFS and EAA. It
can be observed that both methods determine the defect depth
at layer 2 correctly, whereas AFS reconstructs a size of
\( 7 \text{ mm} \times 10 \text{ mm} \times 2 \text{ mm} \) and EAA of \( 11 \text{ mm} \times 2 \text{ mm} \times 2 \text{ mm} \),
respectively.

![Fig. 2. Results of the goal function scan based on AFS and EAA for a cuboidal defect with \( x \)- and \( y \)-extension (12\times2) mm² at the depth 2 mm: The minimal aNRMSE and its corresponding estimated defect extension (\( L_d \times W_d \)) mm² are shown for each layer. The correct defect depth (layer 2) has been found for both forward solutions, whereas the EAA estimates the defect shape more accurately.](image)

The comparison of AFS and EAA for LFE based on a goal
function scan shows that correct defect depth can be estimated
with both methods, whereas a better shape reconstruction can
be achieved by using the EAA. Current work focuses on the
application of the EAA to measurement data.

IV. CONCLUSION

The comparison of AFS and EAA for LFE based on a goal
function scan shows that correct defect depth can be estimated
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application of the EAA to measurement data.

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